Magnetotransport in a parabolic channel exposed to a periodically modulated magnetic field



1998 J. Phys.: Condens. Matter 10 10565

(http://iopscience.iop.org/0953-8984/10/47/008)

J. Phys.: Condens. Matter **10** (1998) 10565–10576. Printed in the UK PII: S0953-8984(98)95023-8

# Magnetotransport in a parabolic channel exposed to a periodically modulated magnetic field

Jian-Ping Peng

Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany†

Received 10 June 1998

**Abstract.** We have studied the energy spectrum and magnetotransport of a parabolic one-dimensional channel in the presence of periodic modulations in both magnetic field and the electrostatic potential. The periodic modulations change the sub-band energy–wave-vector relation and produce mini-gaps at specific wave-vectors. The gaps exist even if there is only modulation in the magnetic field. For the cases where the two kinds of modulation are in phase or out of phase, we give the explicit expressions for the values of the gaps. In the ballistic transport regime, the conductance is expected to remain quantized and to evolve in a simple manner as a function of the Fermi energy. Starting from linear response theory, the transport properties are studied numerically, taking into account the impurity scattering. Calculations related to magnetic field modulation show that a series of cusps in the conductance are produced, whose magnitude depends strongly on the degree of impurity scattering. The structure of these cusps can be explained in terms of the energy band structure.

## 1. Introduction

In recent years, the physics of electrons subjected to magnetic fields which are inhomogeneous on the nanometre scale has attracted much attention. In two dimensions (2D), the magnetotransport of electrons in the presence of a weak periodic modulation of the magnetic field has been studied theoretically [1–4]. The results obtained are interesting, and some of them have now been verified experimentally with the advances in the techniques used to achieve the desired inhomogeneous magnetic field profile [5, 6]. The magnetic field modulation of 2D electron gas can be realized by integration of the heterostructure with strips of magnetic material or superconducting films. However, there is difficulty in observing the effects caused by quantum structures of inhomogeneous magnetic fields such as magnetic quantum steps, barriers, and magnetic wells [5–7]. Firstly, it is not easy to realize experimentally the large magnetic field needed if one is to observe a convincing effect of bound or scattered states. Secondly, the deposition of magnetic materials or superconducting films always introduces strains, which will have an effect on the electric potential beneath them. Although a much stronger magnetic field could be alternatively achieved by bending the structure [8], such a structure seems to be more complicated when it is considered in the study of electrons in the ballistic regime. In this paper we will study theoretically the properties of one-dimensional (1D) electronic systems in the presence of 1D periodic modulations in both magnetic field and electrostatic potential.

† Mailing and permanent address: Department of Applied Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China.

 

Transport in 1D systems has itself been extensively investigated in the past few years. Most of the work concentrated on tiny structures in which the electron transport is ballistic and the electron motion is governed by quantum mechanics. An important example of an interesting phenomenon is the quantization of the conductance in units of $2e^2/h$ in the linear response regime [10]. For longer channels, the electron transport can no longer be ballistic, because the scattering from the imperfection plays an important role. Calculations based on the Boltzmann equation for a system with discrete sub-bands and a discontinuous density of states predicted a so-called quantum-size effect, i.e., discontinuity in the transport coefficients as the Fermi energy or the width of the channel of the 1D system is changed [11]. Unfortunately, the discontinuities observed experimentally have not been as sharp as predicted, and in some cases discontinuities have not been observed at all [12]. Such disagreements led to the development of a transport model based on Green function theory in which scattering caused by impurities is taken into account [13, 14]. It has been shown that the broadening of the electronic states ignored in the Boltzmann theory results in a general smearing out of the quantum-size effect. Although the calculations considered only the elastic scattering from randomly distributed $\delta$-function impurities, the results were shown to reduce to those of Boltzmann theory in the low-impurity limit; the effect of broadening becomes more important as the impurity concentration increases, and the quantum-size effect will disappear when the impurity concentration is high enough.

In this paper we study, in section 2, the sub-band structure of a one-dimensional channel in the presence of a periodic modulation in magnetic field and its consequences for the electron transport in the ballistic regime. In section 3, we use the Green function approach to calculate the magnetotransport of 1D channels when the effects of impurity scattering are taken into consideration. Numerical results for the corrections to the electrical conductivity due to the periodic modulation are presented in section 4. A brief summary and comments are given in section 5.

## 2. Transport in the ballistic regime

The one-dimensional electron system is defined in the two-dimensional electron system in the $xy$-plane by adding a parabolic confinement in the $y$-direction. A magnetic field of strength $B_0$ is applied in the $z$-direction. If we chose $\boldsymbol{A} = (-B_0 y, 0, 0)$ as the gauge for the vector potential, then the Hamiltonian for a electron with effective mass $m$ is

$$\hat{H}_0 = [(P_x - eB_0 y)^2 + P_y^2]/2m + m\Omega^2 y^2/2 \tag{1}$$

where the second term comes from the parabolic confinement whose strength is indicated by the characteristic frequency $\Omega$. The solution of the Hamiltonian is now well known [16]. The energy of the system is of the type

$$E_n = \hbar(\omega_c^2 + \Omega^2)^{1/2}(n + 1/2) + \frac{P_x^2}{2m}\frac{\Omega^2}{\omega_c^2 + \Omega^2} \tag{2}$$

with $\omega_c = eB_0/m$ being the cyclotron frequency, depending on the uniform magnetic field $B_0$. The corresponding wave-function is of the form

$$\Psi_n(x, y) = \psi_n(y - y_0)\exp(\mathrm{i}P_x x/\hbar) \tag{3}$$

where the function

$$\psi_n(y - y_0) = N_n H_n\left(\frac{y - y_0}{a_0}\right)\exp\left[-\frac{1}{2}\left(\frac{y - y_0}{a_0}\right)^2\right] \tag{4}$$

is the wave-function of an oscillator with magnetic length $a_0 = [\hbar/\{m(\omega_c^2 + \Omega^2)^{1/2}\}]^{1/2}$ and centred at $y_0 = [P_x/(eB_0)]\omega_c^2/(\omega_c^2 + \Omega^2)$.

If the channel is further modulated by a periodic magnetic pattern, then the Hamiltonian of the system becomes more complicated. To our knowledge, there is no experiment described in the literature that is concerned with the 1D electron gas modulated by a periodic magnetic field, so we present here only theoretical calculations. Judging by the success in achieving electric or magnetic modulation in the 2D electron gas [5–7], the system studied here may be realized in the near future with the improvement of experimental techniques.

When a modulation in the magnetic field is present, there is a variation in space of the magnetic field. From experience with experiments involving magnetic modulation in 2D electron gases, an important point is apparent. Deposition of magnetic or superconducting materials on the surface exerts mechanical stress on the underlying semiconductor, and leads to an electrostatic modulation in addition to the magnetic one [5–7]. We consider first the simplest case in which the periodic modulation leads to only one sine or cosine component in space. We assume that the magnetic field can be written in the form

$$B_z = B_0 + B_1 \sin(Kx) \tag{5}$$

where $B_0$ represents the average magnetic field, $B_1$ the amplitude of the modulation, and $K = 2\pi/a$, with $a$ being the period in real space. At the same time, there should be a periodic modulation of the electrostatic potential, which could be applied in a direct way or caused by a side effect such as the stress induced when magnetic modulation is realized in an experiment. It is further assumed that, apart from a constant, the electric modulation has the form

$$V(x) = V_E \sin(Kx + \phi) \tag{6}$$

where $\phi$ is the phase difference between the electric and magnetic modulations.

When effects arising from spin are neglected, the general Hamiltonian operator of the system is then

$$\begin{aligned}\hat{H} &= \frac{P^2}{2m} + \frac{e}{2m}(\boldsymbol{P}\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\boldsymbol{P}) + \frac{e^2A^2}{2m} + V(x) \\ &= \frac{P^2}{2m} + \frac{e}{m}\boldsymbol{A}\cdot\boldsymbol{P} - \frac{\mathrm{i}e\hbar}{2m}\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{e^2A^2}{2m} + V(x).\end{aligned} \tag{7}$$

We take the vector potential to be of the form

$$\boldsymbol{A} = (-B_0 + B_1 \sin(Kx))(y, 0, 0) \tag{8}$$

where the total Hamiltonian can be separated into two parts:

$$\hat{H} = \hat{H}_0 + H_\Delta. \tag{9}$$

Here $H_0$ is the Hamiltonian for a homogeneous system as described by (1). The term

$$\begin{aligned}H_\Delta = &-(eB_1P_xy/2m)\sin(Kx) + (\mathrm{i}e\hbar KB_1y/2m) \\ &\times\ \cos(Kx) + (e^2B_0B_1y^2/m)\sin(Kx) + V_E\sin(Kx+\phi)\end{aligned} \tag{10}$$

arises from modulation effects. For such a Hamiltonian, it is in principle difficult to obtain any exact analytical solution, now that it depends on both $x$ and $y$. To find the solution, firstly we assume that the term describing the effects of modulation is small compared with that for the homogeneous field, and consequently the former can be regarded as a perturbation of the latter. Secondly, the variable $y$ in (10) is replaced by its expectation value in the unperturbed states described above. That is to say, we make use of the replacements

$$\begin{aligned} \langle y \rangle &= y_0 \\ \langle y^2 \rangle &= y_0^2 + a_n^2(n+1/2) \end{aligned} \tag{11}$$

in (10). As a result, the following equation is obtained:

$$H_\triangle = \left( \frac{\omega_c^4}{(\omega_c^2+\Omega^2)^2} - \frac{\omega_c^2}{\omega_c^2+\Omega^2} \right) \sin(Kx) \frac{B_1}{B_0} \frac{P_x^2}{m} + \left( \frac{B_1}{B_0} \frac{\omega_c^2}{\omega_c^2+\Omega^2} \cos(Kx) \right) \frac{\mathrm{i}\hbar K P_x}{2m} \\ + V_H(n)\sin(Kx) + V_E \sin(Kx+\phi) \tag{12}$$

where

$$V_H(n) = e^2 B_0 B_1 \left[ \frac{\hbar}{m(\Omega^2+\omega_c^2)^{1/2}} \right] (n+1/2) \tag{13}$$

which depends on the sub-band index $n$. The Hamiltonian is now reduced to a one-dimensional form.

In a real experiment, the shape of the deposited magnetic or superconducting material is most probably symmetric with respect to the two directions along the wire. So we have in fact only two cases, $\phi = 0$ and $\phi = \pi/2$, that need consideration. In the case where $\phi = 0$, i.e., when the electric modulation is in phase with the magnetic one, the Hamiltonian in (9) reads

$$\hat{H}(x) = (A - 2B\sin(Kx)) \frac{\hat{P}_x^2}{2m} + 2\mathrm{i}C(\cos(Kx))P_x + (V_E + V_H(n))\sin(Kx). \tag{14}$$

In the case where $\phi = \pi/2$, i.e., when the electric modulation is out of phase with the magnetic one, the Hamiltonian in (9) reads

$$\hat{H}(x) = (A - 2B\sin(Kx)) \frac{\hat{P}_x^2}{2m} + 2\mathrm{i}C(\cos(Kx))P_x + V_E\cos(Kx) + V_H(n)\sin(Kx). \tag{15}$$

In the above two equations, we have made use of the following notation:

$$\begin{aligned} A &= \Omega^2/(\Omega^2+\omega_c^2) \\ \lambda &= 1 - A \\ B &= \lambda(1-\lambda)B_1/B_0 \\ C &= \hbar K \lambda B_1/4mB_0. \end{aligned}$$

When the length of the wire $L$ is so long that the length is much larger than the period $a$, there is translational invariance along the direction of the wire. We look for a solution for the wave-function following Bloch's theorem:

$$\Psi_{k,n}(x,y) = u_{k,n}(x)\psi_n(y-y_0)\exp(\mathrm{i}kx) \tag{16}$$

where

$$u_{k,n}(x) = u_{k,n}(x+a).$$

Note that the wave-vector $k$ is a good quantum number. Just as for a 1D periodic electric potential, the wave-function depending on $x$ may be expressed as a Fourier series summed over all possible values of the wave-vector. The outcome of this is the central equation, a set of algebraic equations derived from (14) and (15). We write down the matrix of coefficients determined from five successive equations of the central equation. The condition for a non-trivial solution of the coefficients is [9]

$$\det\begin{pmatrix} \gamma(2)-E_k & \alpha(2) & 0 & 0 & 0 \\ \beta(1) & \gamma(1)-E_k & \alpha(1) & 0 & 0 \\ 0 & \beta(0) & \gamma(0)-E_k & \alpha(0) & 0 \\ 0 & 0 & \beta(-1) & \gamma(-1)-E_k & \alpha(-1) \\ 0 & 0 & 0 & \beta(-2) & \gamma(-2)-E_k \end{pmatrix} = 0 \qquad (17)$$

where $j$ is an integer, $\gamma(j) = A\hbar^2(k+jK)^2/2m$, and the parameters $\alpha$ and $\beta$ are respectively defined as

$$\alpha(j) = \begin{cases} \mathrm{i}[B\hbar^2(k+jK)^2/2m + C\hbar(k+jK) - (V_E + V_H(n))/2] & \text{for } \phi = 0 \\ V_E/2 + \mathrm{i}[B\hbar^2(k+jK)^2/2m + C\hbar(k+jK) - V_H(n)/2] & \text{for } \phi = \pi/2 \end{cases}$$

and

$$\beta(j) = \begin{cases} -\mathrm{i}[B\hbar^2(k+jK)^2/2m - C\hbar(k+jK) - (V_E + V_H(n))/2] & \text{for } \phi = 0 \\ V_E/2 - \mathrm{i}[B\hbar^2(k+jK)^2/2m - C\hbar(k+jK) - V_H(n)/2] & \text{for } \phi = \pi/2. \end{cases}$$

Please note that the above matrix reduces gradually to the standard form for a periodic electric potential if the amplitude of the magnetic field modulation decreases. We further proceed to assume that only two sets of coefficients are important when a wave-vector state is in the vicinity of the Brillouin zone boundary, determined by the periodicity $a$. It is found that the gap centred at $|k| = K/2$ is

$$\Delta_n = \begin{cases} \left| \dfrac{\hbar^2 K^2}{4m} \dfrac{B_1}{B_0}(1-A^2) - (V_E + V_H(n)) \right| & \phi = 0 \\ \left\{ V_E^2 + \left[ \dfrac{\hbar^2 K^2}{4m} \dfrac{B_1}{B_0}(1-A^2) - V_H(n) \right]^2 \right\}^{1/2} & \phi = \dfrac{\pi}{2} \end{cases} \qquad (18)$$

which is a function of the sub-band index $n$. In an idealized situation where there is only the magnetic field modulation and no side effect is present, i.e., $V_E = 0$, the gap becomes simplified:

$$\Delta_n = \left| \frac{\hbar^2 K^2}{4m} \frac{B_1}{B_0}(1-A^2) - V_H(n) \right|. \qquad (19)$$

This magnetic-field-induced energy gap is proportional to the amplitude of the modulation $B_1$. The formation of these gaps has important implications for the transport in the system. It is well known that there is a one-to-one correspondence between the index of a quantized conductance plateau and the number of transverse bands with positive group velocity in the electrically modulated 1D systems. This conclusion was drawn from a comparison between the calculated conductance and the band structure, and even holds in the presence of an applied homogeneous magnetic field [18]. For a structure with several magnetic barriers or wells, we have performed numerical calculations of the same kind using the transfer-matrix method. We have come to the same conclusion—that the conductance is quantized, and the number of quantized plateaus is equal to the number of transverse modes with positive group velocity in the structure. The effects of the magnetic field barriers or wells is to cause inter-mode scattering.

The relationship between the band structure and the conductance in the ballistic regime is shown schematically in figure 1. In the figure, we have assumed that there is only one component of modulation present in the system, and therefore the gaps are opened only in the vicinity of the wave-vector $|k| = K/2$. The gaps shown are only to guide the

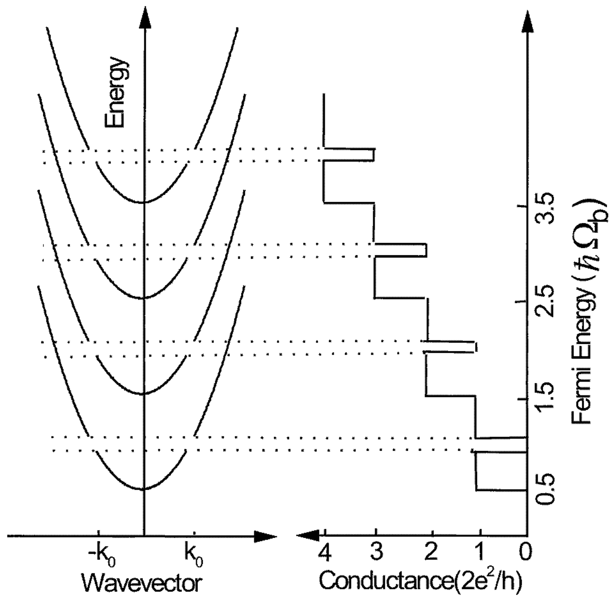


**Figure 1.** A schematic view of the correspondence between the conductance and the Fermi energy of the electrons for a parabolic quantum wire in the presence of a perpendicular magnetic field. The gaps are opened as a result of modulations of the magnetic field and electric potential. In this figure, it is assumed that there is only one single component of sine or cosine modulation, characterized by $k_0 = \pi/a$.

eyes though; their values vary with the different sub-bands. In principle, there should be changes in the curvature of the energy–momentum relation in the vicinity of a gap; these are neglected in the figure. This seems to be important, because of the divergence of the density of states near a gap. According to a formalism starting from Boltzmann's transport equation, such a divergence leads to a large discontinuity in the conductance. However, the experimental search for the conductance discontinuity will not be successful. The reason for this is that the Boltzmann equation fails to provide a correct description when the Fermi energy is near the sub-band edge. The best way to see this is to consider the effect of scattering from impurities using the Green function approach, and this will be discussed in the next section. We believe that the discontinuity is finite even in the limit of an impurity-free system, as demonstrated by the quantum mechanical calculations for tiny structures. If scattering from imperfections is to be neglected, the quantized conductance is determined by the number of transverse modes with positive group velocity, not by the exact value of the velocity.

## 3. Effects of elastic scattering

In the idealized model discussed in the above section, we have ignored all effects arising from all kinds of scattering, i.e., the electron transport is assumed to be ballistic. This is a good approximation when we are dealing with a high-purity sample, or, more precisely, when the sample length is much less than the electron mean free path. However, for a longer wire, the effects due to scattering become so important that the picture of quantized conductance breaks down. In a real system one should take into account such effects as impurity scattering, surface roughness, and variations in the widths of the wire. Recent theoretical work on these subjects has shown that an account of elastic scattering due to weak disorder is enough to show the general picture [11–14]. The simple case considered is that in which electrons are scattered by randomly distributed $\delta$-function impurities described by the potential

$$V_i(x, y) = a_I \sum_g \delta(\boldsymbol{r} - \boldsymbol{R}_g) \tag{20}$$

where $a_I$ defines the strength of the scatterer and $\boldsymbol{R}_g$ represents random sites. These scatterers cause lifetime broadening of the multiply occupied 1D sub-band of the system. When there is no magnetic field, the lifetimes are found to differ from sub-band to sub-band, independently of the wave-vector in the wire direction. The scattering is stronger when the energy is near the edge of a sub-band, and thus there is a peak in the density of states. These peaks in the density of states, unlike the infinite jumps predicted by a simple consideration, depend strongly on the strength and density of the scatterers, and are consistent with experimental observations [11–14]. When there is an magnetic field applied perpendicular to the wire, the imaginary part of the self-energy of a parabolic 1D wire is determined by the self-consistent set of equations [16, 17]

$$\Gamma_{n,k}(E) = \sum_{n'} \int \frac{\mathrm{d}k'}{2\pi} \overline{|\langle n, k|V_i|n', k'\rangle|^2} \frac{\Gamma_{n',k'}(E)}{[E - E_{n'}(k')]^2 + \Gamma^2_{n',k'}(E)}. \tag{21}$$

Here the configurational average for a parabolic confinement is

$$\overline{|\langle n, k|V_i|n', k'\rangle|^2} = a_I^2 N_i \int_{-\infty}^{\infty} \mathrm{d}y \;\psi_n^2(y - y_0)\psi_{n'}^2(y - y_0') \tag{22}$$

where $N_i$ is the density of impurities. The real part of the self-energy is as usual believed to be unimportant, and is not considered in the present context. The degree of scattering is measured by the parameter

$$p = a_I^2 N_i \tag{23}$$

which means that a stronger scattering is expected when there are more impurity scatterers interacting more strongly with electrons. In the calculations shown below, we have chosen a specific value of $p = p_0$ such that the solution for the level broadening $\Gamma$ is of the same order of magnitude as the characteristic level separation $\hbar\Omega$. The larger the value of $p$, the stronger the scattering effects.

The magnetic field leads to a reduction of the scattering matrix elements for large momentum transfer, and therefore the lifetimes depend not only on the sub-band, but also on the wave-vector in the wire direction [14, 16]. Furthermore, the peaks in the total density become more widely spaced, and increase in magnitude as the magnetic field is increased. A wider space between peaks can be easily explained by the energy spectrum (2). Increase in the peak height is very important from an experimental point of view. A sharpened feature would lead to a greater possibility of observing structure owing to sub-band structure, since sub-band structure may be masked at zero magnetic field by other scattering mechanisms that have been ignored. This is why the experiments in reference [17] and reference [16] succeeded in observing the structure due to sub-band depopulation by varying the magnetic field rather than the Fermi energy.

The conductivity in one dimension is [12–15]

$$\sigma = -e^2\hbar \sum_n \int \frac{\mathrm{d}k}{2\pi} \; v_k^2 \int \frac{\mathrm{d}E}{2\pi} \; A_{nn}^2(k, E) \frac{\partial f(E)}{\partial E} \tag{24}$$

where $A_{nn}(k, E)$ is the spectral function, and $v_k = \partial E(k)/\partial k/\hbar$ is the group velocity of the particle. This formula, derived under the condition of zero temperature and in the linear response regime, is in fact valid not only for free electrons, but also for quasiparticles.

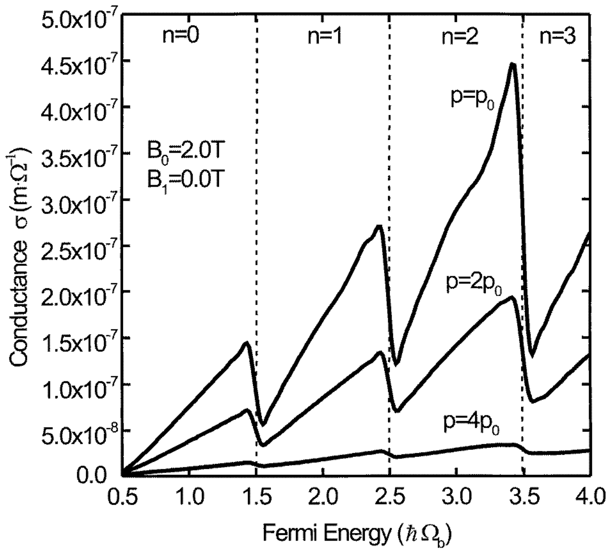


**Figure 2.** The conductivity versus the Fermi energy for a normal wire under an applied magnetic field, but where there is no modulation of any kind. The dashed lines are drawn to indicate the number of occupied sub-bands $n$. From top to bottom, the scattering effects become stronger, as indicated by the parameter $p$. The meanings of $n$ and $p$ are the same in the figures below.

We have numerically solved the self-consistent equations (21) for the lifetime broadening $\Gamma$ for up to four occupied sub-levels. The conductivity is plotted as a function of the Fermi energy in figure 2. The parabolic confinement is characterized by the energy interval $\hbar\Omega = 3$ meV, and there is a magnetic field of $B_0 = 2$ T. The parameters are so chosen that the parabolic confinement and magnetic field terms are of comparable magnitude, and thus the electron states are best described as hybrid magnetoelectric states. We have scaled the Fermi energy in terms of the separation between the hybrid sub-levels, which is related to the confinement and the magnetic field by

$$\hbar\Omega_b = (\omega_c^2 + \Omega^2)^{1/2}. \tag{25}$$

As the Fermi energy is increased, more sub-bands will be occupied. This is indicated in the figure by the dashed lines, and the number $n$ is the index of the occupied sub-band. When a new sub-band is opened, there is a sudden drop in the conductance, which is attributed to the reduction of the dimensions due to the parabolic confinement and is the so-called quantum-size effect. The calculations were performed for different degrees of scattering. The behaviours of the lifetime are found to be similar to those found in reference [16], in which the authors used a square-box model for the wire. When the scattering is weaker, the jump in the conductance is larger. The structure of the conductance jump will be removed if the scattering is strong enough, which is in agreement with the numerical calculations made by Kearney and Butcher [13]. The impurity scattering makes it difficult to observe the predicted structure experimentally by varying the Fermi energy.

A better way to observe the conductance change due to 1D confinement is to use the process of electron depopulation; i.e., at relatively constant Fermi energy, varying the

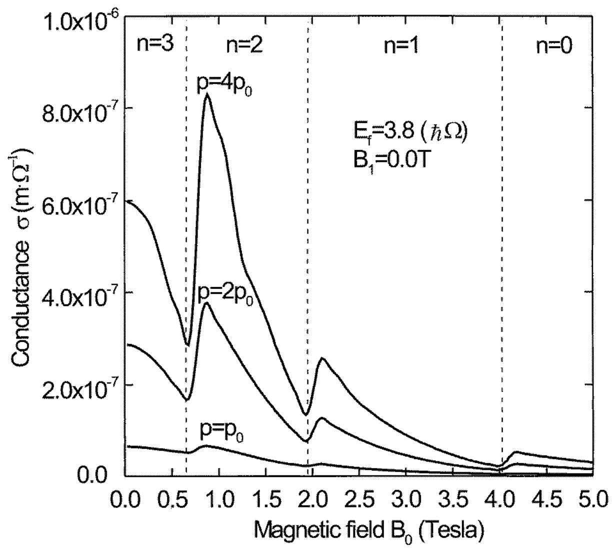


**Figure 3.** The conductivity versus the magnetic field for a normal wire at constant Fermi energy without modulation. This figure is qualitatively consistent with the experiment in reference [14], where the magnetoresistance of a channel was measured.

applied magnetic field will cause electrons to fill or to empty a sub-band as in the experiment of Berggren *et al* [16]. The numerical results are shown in figure 3. The Fermi energy is kept constant, $E_F = 3.8\hbar\Omega$, and the magnetic field varies from a low value continuously to a higher one. At low magnetic field, there are four initially occupied sub-bands. As the magnetic field is increased, the sub-level separation increases according to equation (25), resulting in the process of electron depopulation. Finally, in the high-field limit, only the lowest sub-band is occupied by electrons. This seems to be the inverse of the process occurring with increasing Fermi energy in figure 2. It can be seen that when a sub-band becomes empty, there is a jump—not a drop—in the conductivity. In a real sample at finite temperature, the sudden jump or drop will evolve into a continuous rise or decrease—i.e., oscillation of some kind in the conductivity—because there is always a small proportion of the electrons that are excited into higher energy states. Experiments have observed this kind of oscillation in the magnetoresistance of 1D channels, and have shown that at a lower field the structure associated with depopulation of sub-bands may be masked by quantum fluctuations [16].

## 4. Corrections to the conductance due to magnetic modulation

As discussed in section 2, a periodic modulation in the magnetic field or electric potential causes mini-gaps at specific wave-vectors and produces structure in the conductance. If the two kinds of modulation have the same period, the values of the mini-gaps can be determined analytically by using equation (18) under some reasonable conditions. As far as conductance is concerned, the most important consequence for a wave-vector falling into a gap region is that the group velocity of the electron becomes zero, and thus this state does not contribute to the conductance. Because all of the formalism in equations (21) and (22) is in terms of the good quantum numbers $k$ and $n$, it can also be applied to the case in

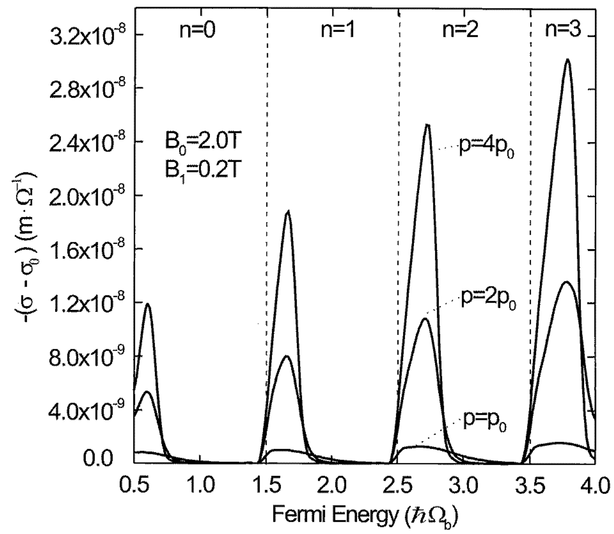


**Figure 4.** The correction to the conductivity due to magnetic modulation. The curves are plotted as functions of the Fermi energy for different degrees of disorder. The modulation is taken to be of the form $B_z = B_0 + B_1 \sin(Kx)$, and no electric potential modulation is present.

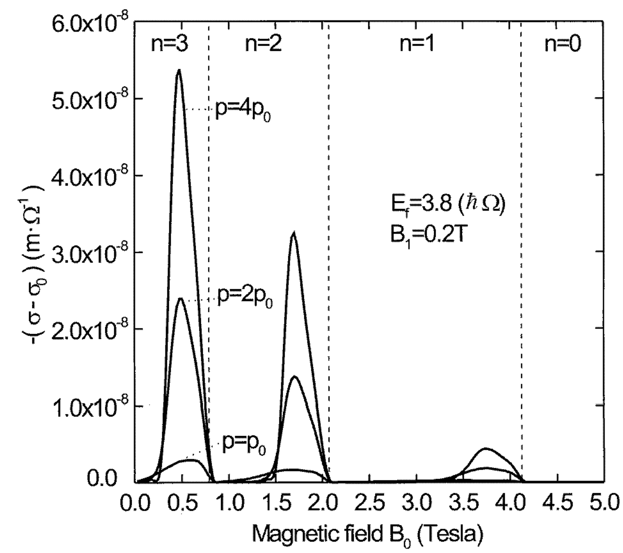


**Figure 5.** The correction to the conductivity due to magnetic modulation. The curves are plotted as functions of the average magnetic field for different degrees of disorder. The modulation is taken to be of the form $B_z = B_0 + B_1 \sin(Kx)$, and it is assumed that there are no side effects, and therefore that there is no electric potential modulation.

which there is modulation as well.

Figure 4 and figure 5 show the changes in conductance that occur when a magnetic field modulation of the form $B_z = B_0 + B_1 \sin(Kx)$ is added, as functions of the Fermi energy and the average magnetic field respectively. Here $\sigma_0$ is the one-dimensional conductivity as in figure 2 or figure 3, and $\sigma$ is the conductivity when the modulation is added to the system. In the numerical calculations we have assumed that the confinement strength is $\hbar\Omega = 3$ meV, the period of the modulation is $a = 0.5$ $\mu$m, and the amplitude of the modulation is $B_1 = 0.2$ T. Several plots are again presented for different degrees of scattering from impurities. No electric potential modulation is included in the calculations here, because there is not yet a simple description available for the deformation or the stress-induced effects on the electrostatic potential. We believe that the general picture will not change drastically, since the formalism is general, and can include effects from both the magnetic field modulation and the possible modulation in the electrostatic potential. Furthermore, the properties of a 1D system such as an energy band structure and its effect on the transport exposed to a periodic modulation is now understood more clearly, and consequently the emphasis will be kept on the effects arising from magnetic field modulation.

It can be seen that the main feature of the effect of the addition of a periodic magnetic field modulation is the reduction in the conductivity in specific regions of the Fermi energy or magnetic field, where gaps are opened. Theoretically there is a one-to-one correspondence between these regions and the centres of the mini-gaps in the energy spectrum. This is not a surprise, because the results in figure 4 are in fact the counterpart of those in figure 1 for when the effects of impurity scattering are taken into account. However, there are some differences between them. Firstly, the effect is now sub-band dependent, which is demonstrated in figure 4 or figure 5 by a higher peak value for a larger sub-band index $n$. Secondly, the widths of the peaks in figure 4 are wider than the values of the respective mini-gaps determined by equation (19). The effect on the conductance spreads over a wider region for a stronger impurity scattering, besides which the peaks are significantly reduced.

Since in a real sample with modulation, the conductance may be viewed as a superposition of figures 2 and 4 or figures 3 and 5, it is interesting that we find that the peak values of the correction to the conductance due to modulation are in general one order of magnitude smaller than the original conductance. For the parameters used in our calculations, the following relation is obtained:

$$\sigma_0/(\sigma_0 - \sigma) \approx 10\text{–}20 \tag{26}$$

which is almost independent of the strength of the impurity scattering. This may be important from the point of view of the experimentalist. However, to compare with experiments we have to take into consideration in our calculations the possible electrostatic potential modulation when a magnetic field modulation is realized.

## 5. Summary

We have studied the energy spectrum and conductance of a model 1D electron system, in the presence of sine or cosine periodic modulations in a magnetic field and an electrostatic potential. Under the idealized condition of ballistic transport, the conductance should remain quantized and evolve in the manner shown in figure 1 as a function of the Fermi energy. The periodic modulations change the sub-band energy–wave-vector relation and produce mini-gaps at specific wave-vectors, and therefore induce cusps in the conductance. The main features of such a relationship do not change drastically when the scattering from impurities is taken into account. Both the quantum-size effect and the correction of the conductance due to modulations are sensitive to the impurity density and the strength of the scattering potential, and will be drastically reduced if the scattering becomes stronger. These conclusions, based on calculations for zero temperature and in the linear response regime, should remain valid in describing transport in a real channel at low temperature. For a real system with a general periodic modulation, what we need to do is to express the potential and magnetic field in Fourier series. Every single component of the modulation makes a contribution, as shown in the present paper.

## Acknowledgments

The author would like to thank Professor Dr U Roessler at the University of Regensburg, Germany, for many helpful discussions, and thanks the Alexander von Humboldt Foundation, Germany, for a fellowship.

## References

[1] Xue D P and Xiao G 1992 *Phys. Rev.* B **45** 5986
[2] Peeters F M and Vasilopoulos P 1992 *Phys. Rev.* B **46** 4667
Peeters F M and Vasilopoulos P 1993 *Phys. Rev.* B **47** 1466
[3] Peeters F M and Matulis A 1993 *Phys. Rev.* B **48** 15 166
[4] Gerhards R R 1996 private communication
[5] Carmona H A, Geim A K, Nogaret A, Main P C, Foster T J and Henini M 1995 *Phys. Rev. Lett.* **74** 3009
[6] Ye P D, Weiss D, Gerhards R R, Seeger M, von Klitzing K, Eberl K and Nickel H 1995 *Phys. Rev. Lett.* **74** 3013
[7] Izawa S, Katsumoto S, Endo A and Iye Y 1995 *J. Phys. Soc. Japan* **64** 706
[8] Foden C L, Leadbeater M L, Burroughes J H and Pepper M 1995 *J. Phys.: Condens. Matter* **6** L127
Leadbeater M L, Foden C L, Burke T M, Burroughes J H, Grimshaw M P, Ritchie D A, Wang L L and Pepper M 1995 *J. Phys.: Condens. Matter* **6** L307
[9] Kittel C 1986 *Introduction to Solid State Physics* (New York: Wiley)
[10] Xu Hongqi 1994 *Phys. Rev.* B **50** 8469 and references therein
[11] Kearney M J and Butcher P N 1986 *J. Phys. C: Solid State Phys.* **19** 5429
[12] Sernelius B E, Berggren K-F, Tomak M and McFadden C 1985 *J. Phys. C: Solid State Phys.* **18** 225
[13] Kearney M J and Butcher P N 1987 *J. Phys. C: Solid State Phys.* **20** 47
[14] Cantrell D G and Butcher P N 1985 *J. Phys. C: Solid State Phys.* **18** 5111
[15] Ferry D K and Jacoboni C (ed) 1992 *Quantum Transport in Semiconductors* (New York: Plenum)
[16] Berggren K-F, Roos G and van Houten H 1988 *Phys. Rev.* B **37** 10 118
[17] Berggren K-F, Thornton T J, Newson D J and Pepper M 1986 *Phys. Rev. Lett.* **57** 1769
[18] Leng M and Lent C S 1985 *Phys. Rev.* B **50** 385